Appendix Q

Removal of Overflow Problems in the Intensity Formulas

We start looking at the homogeneous part of eqn. 8.67 for the upward intensity, which we rewrite as (using $k_{-jp} = -k_{jp}$)

$$I_p^+(\tau,\mu) =$$
$$\sum_{n=p}^{L}\sum_{j=1}^{N}\{C_{-jn}\frac{\tilde{g}_{-jn}(+\mu)}{1-k_{jn}\mu}e^{-[-k_{jn}\tau_{n-1}+(\tau_{n-1}-\tau)/\mu]} - e^{-[-k_{jn}\tau_n+(\tau_n-\tau)/\mu]}$$
$$+ C_{+jn}\frac{\tilde{g}_{+jn}(+\mu)}{1+k_{jn}\mu}e^{-[k_{jn}\tau_n+(\tau-\tau_n)/\mu]} - e^{-[k_{jn}\tau_{n-1}+(\tau-\tau_{n-1})/\mu]}\}. \quad \text{(Q.1)}$$

Introducing eqns. 8.57 into eqn. 8.67, we find

$$I_p^+(\tau,\mu) = \sum_{n=p}^{L}\sum_{j=1}^{N}\{C'_{-jn}\frac{\tilde{g}_{-jn}(+\mu)}{1-k_{jn}\mu}E'_{-jn}(\tau,+\mu)$$
$$+C'_{+jn}\frac{\tilde{g}_{+jn}(+\mu)}{1+k_{jn}\mu}E'_{+jn}(\tau,+\mu)\} \quad \text{(Q.2)}$$

where

$$E'_{-jn}(\tau,+\mu) = \exp\left[-(k_{jn}\Delta\tau_n + \delta\tau/\mu)\right] - \exp\left[-(\tau_n-\tau)/\mu\right] \quad \text{(Q.3)}$$

with

$$\begin{cases} \Delta\tau_n = \tau_n - \tau_{n-1}, & \delta\tau = \tau_{n-1} - \tau \quad \text{for } n < p \\ \Delta\tau_p = \tau_p - \tau, & \delta\tau = 0 \quad \text{for } n = p \end{cases}, \quad \text{(Q.4)}$$

$$E'_{+jn}(\tau,+\mu) = \exp\left[-(\tau_{n-1}-\tau)/\mu\right] - \exp\left\{-\left[k_{jn}(\tau_n-\tau_{n-1}) + (\tau_n-\tau)/\mu\right]\right\} \quad \text{(Q.5)}$$

for $n > p$ and

$$E'_{+jp}(\tau,+\mu) = \exp[-k_{jp}(\tau-\tau_{p-1})]$$
$$- \exp\left\{-[k_{jp}(\tau_p-\tau_{p-1}) + (\tau_p-\tau)/\mu]\right\}. \quad \text{(Q.6)}$$

Since $k_{jn} > 0$ for $n = p+1, p+2, \ldots L$ and $\tau_L > \cdots > \tau_{n=p+1} > \tau_{n-1=p} > \tau$ and also $k_{jp} > 0$ and $\tau_{p-1} < \tau < \tau_p$, all the exponentials in eqns. Q.3–Q.6 have negative arguments as they should.

Similarly, by introducing eqns. 8.57 into the homogeneous part of eqn 8.68, we find that the expression for the downward intensity becomes

$$I_p^-(\tau,\mu) = \sum_{n=1}^{p}\sum_{j=1}^{N}\{C'_{-jn}\frac{\tilde{g}_{-jn}(-\mu)}{1+k_{jn}\mu}E'_{-jn}(\tau,-\mu) + C'_{+jn}\frac{\tilde{g}_{+jn}(-\mu)}{1-k_{jn}\mu}E'_{+jn}(\tau,-\mu)\} \tag{Q.7}$$

where

$$E'_{+jn}(\tau,-\mu) = \exp[-(k_{jn}\Delta\tau_n + \delta\tau/\mu)] - \exp[-(\tau-\tau_{n-1})/\mu] \tag{Q.8}$$

with

$$\begin{cases} \Delta\tau_n = \tau_n - \tau_{n-1}, & \delta\tau = \tau - \tau_n \quad \text{for } n < p \\ \Delta\tau_p = \tau_p - \tau_{p-1}, & \delta\tau = 0 \quad \text{for } n = p \end{cases}, \tag{Q.9}$$

$$E'_{-jn}(\tau,-\mu) = \exp[-(\tau-\tau_n)/\mu] - \exp\{-[k_{jn}(\tau_n-\tau_{n-1}) + (\tau-\tau_{n-1})/\mu]\} \tag{Q.10}$$

for $n < p$ and

$$E'_{-jp}(\tau,-\mu) = \exp[-k_{jp}(\tau_p-\tau)] - \exp\{-[k_{jp}(\tau_p-\tau_{p-1}) + (\tau-\tau_{p-1})/\mu]\}. \tag{Q.11}$$

Again, we see that all exponentials involved in the scaled solutions have negative arguments since $k_{jn} > 0$ and $\tau > \tau_n > \tau_{n-1}$ for $n = 1, 2, \ldots, p-1$, and also $k_{jp} > 0$ and $\tau_{p-1} < \tau < \tau_p$. This ensures that fatal overflow errors are avoided in the computations.