Appendix R

Integration of the Planck Function across an Arbitrary Spectral Interval

Our problem in integrating the radiative transfer equation across a spectral interval $\Delta\nu$ is that we must approximate integrals of the form

$$\int_{\Delta\nu} B_\nu \Psi_\nu d\nu \tag{R.1}$$

where Ψ_v is the product of the radiative intensity and possibly other frequency-dependent factors. The radiative intensity is given by the Planck function (§4.4):

$$I_\nu^{BB} = B_\nu(T) \equiv \frac{m_r^2}{c^2} \frac{2h\nu^3}{\left(e^{h\nu/k_B T} - 1\right)}.$$

A sound procedure which conserves energy and is rigorously correct in the limit of zero or infinite absorption, is to approximate this integral as

$$\int_{\Delta\nu} B_v \Psi_\nu d\nu = \frac{\xi_{\Delta\nu}}{\Delta\nu} \int_{\Delta\nu} \Psi_\nu d\nu \tag{R.2}$$

where

$$\xi_{\Delta\nu} = \int_{\nu_1}^{\nu_2} B_\nu d\nu = \xi_{\nu_2} - \xi_{\nu_1} \qquad \text{and} \qquad \xi_\nu = \int_0^\nu B_{\nu'} d\nu'. \tag{R.3}$$

Applying the mean value theorem by pulling B_ν through the integral sign and evaluating it at some wavenumber inside $\Delta\nu$ is a much inferior procedure that neither conserves energy nor gives the correct answer in limiting cases (such as when there is no radiatively active medium above a black surface).

A well-documented numerical procedure for evaluating the Planck function is provided in the DISORT report available at the following web-site:

ftp://climate.gsfc.nasa.gov/pub/wiscombe/Multiple Scatt/

A FORTRAN-77 subroutine for computing the Planck function is also available at this web-site as part of the DISORT code.