Appendix S

Computation of the Normalized Associated Legendre Polynomials

We follow the scheme introduecd by Dave and Armstrong† to compute Λ_l^m, the normalized associated Legendre polynomials.

The only stable recurrence for the Λ_l^m is the one involving its subscript. We take it from Dave/Armstrong Eq. (10) but set $l = k + 1$ and $m = n - 1$ to conform to usual notation

$$\Lambda_l^m(u) = \frac{(2l-1)u\Lambda_{l-1}^m(u) - \sqrt{(l+m-1)(l-m-1)}\Lambda_{l-2}^m(u)}{\sqrt{(l-m)(l+m)}} \qquad (l = m+2, \ldots). \qquad \text{(S.1)}$$

To initialize this recurrence, we need Λ_m^m and Λ_{m+1}^m. Dave and Armstrong express them as multi-step recurrences, but it is possible to derive simpler expressions requiring only a single recursive step. These derivations are provided in the DISORT report that is available at the follwing web-site:

ftp://climate.gsfc.nasa.gov/pub/wiscombe/Multiple Scatt/

A FORTRAN-77 subroutine for computing the Λ_l^m is also available at this web-site as part of the DISORT code.

† Dave, J. V. and B. H. Armstrong, *Computation of High-Order Associated Legendre Polynomials*, J. Quant. Spectros. Radiat. Transfer, 10, 557–562, 1970.